Course Title: Differential Equations

Course Prefix/Number: MAT 242

COURSE-SPECIFIC GRADE CALCULATION
Advanced notification of any changes will be provided to the student.

Three categories of grades will be used to compute your final average.

- Discussion Board/Class Participation – 10%
- Homework – 40%
- Tests – 50% (Proctored Test 10%, Non-Proctored Tests 40%)

EXPLANATION OF SPECIFIC PROCTORED EXAM INFORMATION
For all Mat 242 courses, the Unit 1 Test will be proctored. The students may use a table of integrals and derivatives during the proctored test. Blank scratch paper and a scientific or graphing calculator are allowed. The calculator must not be a part of another device (for example, an app on a smartphone). Scratch paper should be turned in at the end of the exam.

LAB/CLASSROOM SAFETY STATEMENT
Piedmont Technical College Laboratory Safety Statement: Lab Safety Statement (www.ptc.edu/courseinfo/safety.pdf)

Classroom Safety Statement: N/A

COURSE CONTENT OUTLINE
Advanced notification of any changes will be provided to the student.

Modules/Units

Module/Unit 1

Competencies:
INTRODUCTION TO DIFFERENTIAL EQUATIONS AND FIRST ORDER DIFFERENTIAL EQUATIONS

The student will:

- Be familiar with differential equations examples from physics.
- Identify first, second, and higher order differential equations.
- Distinguish and verify explicit and implicit solutions.
- Identify initial value problems and verify a solution.
- Be familiar with the existence and uniqueness theorem for first order Diff. Eq.
- Be able to construct the direction field for a diff. Eq.
- Construct isoclines.
- Use the Method of Euler to approximate the solution to an initial value problem.
- Solve separable equations
- Verify explicit and implicit solutions of differential equations, analyze initial-value problems and the existence and uniqueness nature of solutions with Picard’s Theorem.
- Solve first order linear differential equations with constant and variable coefficients and related initial-valued problems by the integrating factor method.
- Solve separable equations and associated initial-value problems by separation of variables.
- Solve some special cases of first-order Diff. Eq.
- Find solutions of first-order Diff. Eq. by using the integrating factor.
- Identify and solve exact equations.
- Find special integrating factor for non-exact equations.
- Identify and solve some homogeneous equations.
- Solve Bernoulli equations.
- Determine the solution of equations with linear coefficients.
Module/Unit 2

Competencies:
MODELING WITH FIRST ORDER DIFFERENTIAL EQUATIONS AND NUMERICAL METHODS

The student will:
- Solve missing problems
- Work with some population models.
- Use Newton’s law of cooling to solve heating and cooling problems.
- Determine the solution to falling body problems.
- Solve problems involving Kirchhoff’s current and voltage laws.
- Find approximate solutions using different: numerical methods: Euler’s, Trapezoidal scheme, predictor-corrector, and improved Euler’s.
- Compute approximations to some differential equations using higher Order numerical methods: Taylor and Runge-Kutta.

Module/Unit 3

Competencies:
LINEAR SECOND-ORDER EQUATIONS

The student will:
- Find solutions to mass-spring oscillator problems
- Be familiar with the concept of linear independence of functions.
- Find a set of linearly independent solutions for homogeneous linear equations.
- Compute solutions for homogenous linear equations with constant coefficients whose auxiliary equations has complex roots
- Using the method of undetermined coefficients find a solutions to a non-homogeneous linear equations with constant coefficients.
• Using undetermined coefficients and the superposition principle solve non-homogeneous linear equations with constant coefficients.
• Use variation of parameters to find a solution for the non-homogeneous case.

**Module/Unit 4**

Competencies:

**INTRODUCTION TO SYSTEMS AND PHASE PLANE ANALYSIS AND THEORY OF HIGHER ORDER LINEAR DIFFERENTIAL EQUATIONS**

The student will:

• Work with differential operators
• With the elimination method and differential operators, solve a 2x2 linear system of differential equations.
• Find approximate solutions to systems of diff. eq. with Euler’s method.
• Used the vectorized Eulers method to solve higher order diff. eq.
• With the vectorized Runge-Kutta method solve a non-liner diff. eq. system.
• For an autonomous system, be familiar with the concepts of trajectories in the phase plane, phase portrait of the system, critical point, equilibrium solution, stable and unstable equilibrium points, and asymptotically stable critical point.
• Construct the phase portrait for an autonomous system.
• Find the direction field and the phase portrait of an autonomous system.
• Familiar with the different kinds of critical points in the phase plane
• Be familiar with the existence and uniqueness theorem for higher order linear equations.
• Determine the Wronskian of a set of functions
• Compute the general solution of a higher order homogeneous linear diff. eq.
• Be able to determine if a set of solutions in linear independent.
• Understand the connection between the Wronkiand and the linear independence (or dependence) of a set of solutions of a homogeneous linear diff. eq.
• Determine the general solution of some non-homogeneous linear diff. equations of higher order.
• Compute the solution of a higher order homogeneous linear diff. eq. with constant coefficients.
• Find the annihilator of a linear combination of functions
• Use the annihilator method or the undetermined coefficients method to determine particular solutions to some especial non-homogeneous equations.
• Use a generalization of the variation of parameters method to find a particular solution to a non-homogeneous linear dif. eq. with variable coefficients.

Module/Unit 5

Competencies:

THE LAPLACE TRANSFORM

The student will:
• Find Laplace transforms of some elementary functions.
• Be familiar with the properties of the Laplace transform
• Be able to obtain the inverse Laplace transform of a function.
• Use the decomposition in partial fraction to obtain the inverse Laplace transform of a rational function
• With the method of Laplace transform find the solutions to some diff.
• Compute the transform of certain discontinuous functions: unit step, rectangular window, piecewise
• Determine the Laplace transform using the translation in t theorem.
• Find the Laplace transform of a periodic function.
• Use the gamma function to determine the Laplace transform of a negative power.
• Be able to compute the convolution of two functions.
• Be familiar with the properties of the convolution.
• Use the convolution theorem to compute the Laplace transform of a convolution.
• Solve a linear system with constant coefficients using the impulse-response functions.
• Be familiar with the definition of the Dirac Delta function and its Laplace transform.
• Know how the Dirac delta function is used to model some mechanical vibration problems.
• Solve linear systems using the method of Laplace transforms.

Module/Unit 6

Competencies:

SERIES SOLUTIONS OF DIFFERENTIAL EQUATIONS

The student will:

• Approximate the solutions to differential equations with Taylor polynomials.
• Compute the radius for convergence of a power series.
• Use power series to determine the general solution of a linear differential equation.
• Compute the minimum radius of convergence for power series solutions of differential equations.
• Determine derivatives and integrals of power series.
• Be familiar with the definition of analytic function.
• Distinguish between ordinary and singular points of a homogeneous linear diff. eq.
• Determine the singular points of a linear diff. eq.
• Compute the first few terms of a power series general solution
• Determine the general expression for the coefficients of a power series solution
• Find the minimum radius of convergence of a power series solution about a given point.
• Determine the general solution for initial value problems with homogeneous linear diff. eq.’s with analytic coefficients.

Module/Unit 7

Competencies:

MATRIX METHODS FOR LINEAR SYSTEMS

The student will:
• Identify the coefficient matrix of a system of differential equations
• Become familiar with the algebra and calculus of matrices and vectors
• Rewrite a system of differential equations in normal form using matrices.
• Represent solutions of a homogeneous system in terms of linearly independent solutions.
• Represent solutions of a non-homogeneous system in term of linearly independent solutions.